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ESTIMATING THE INFORMATION PARAMETER OF A TWO-WAY TABLE WITH APP--ETC(U)
JAN 78 W H DUMOUCHEL, N ODEN

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WITH APPLICATIONS TO ANIMAL COMMUNICATION

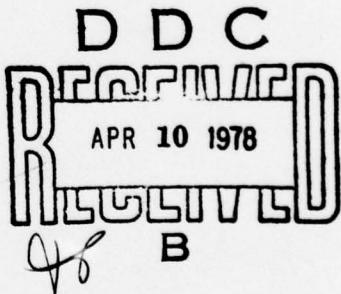
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TECHNICAL REPORT NO. 10
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DEPARTMENT OF MATHEMATICS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASSACHUSETTS

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1. Introduction

Recently a number of studies of animal behavior (Hazlett and Bossert, 1965, 1966; Dingle, 1969, 1972; Altmann, 1965) have been concerned with estimating the dependence between the successive acts of two communicating animals. If one is given $p = \{p_{jk}\}$, the probability that act type k follows act type j , $j = 1, \dots, J$; $k = 1, \dots, K$ where the first act has J possibilities and the second act has K possible types, (often $J = K$), a measure of dependence between successive acts is:

$$I(p) = \sum_j \sum_k p_{jk} \log_2 (p_{jk} / p_j \cdot p_k),$$

where p_j indicates summation with respect to k , and similarly for p_k .

If communication is equivalent to dependence, this quantity may be (and has been) construed to measure the amount of "information" transmitted from one animal to another. Other workers have used I or related measures to assess the information transmitted by the waggle dance of honey bees (Haldane and Spurway, 1954) and the odor trail of fire ants (Wilson, 1962), and to measure dependence between successive phrases in the call of the wood pewee (Chatfield and Lemon, 1970). A good review of the pitfalls involved in the measure of information transfer is given by Cronbach (1955).

In applying this information measure, workers in animal behavior

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usually replace p_{jk} by

$$\hat{p}_{jk} = n_{jk}/n ,$$

where n_{jk} is the total number of times act type k is observed to follow act type j , and $n = n..$ is the total number of observed pairs of adjacent acts.

The usefulness of $I(\hat{p})$ has been impaired by lack of knowledge of its distribution. Although a few workers (Chatfield and Lemon, 1970; Oden, 1977) have noted that, under the null hypothesis of no dependence, $I(\hat{p})$ is closely related to G^2 , the likelihood ratio test of independence in a two-way table (and is therefore roughly related to Pearson's χ^2) by the equation:

$$\begin{aligned} G^2 &= 2 \sum_j \sum_k n_{jk} \log_e (n_{jk} n.. / n_{j.} n_{.k}) , \\ &= 2n \log_e 2 I(\hat{p}) \\ &\doteq 1.39n I(\hat{p}) , \end{aligned}$$

tests of alternate hypotheses have not been performed. Since nonzero values of I are of interest, for example, in comparing inter-act dependence in various species, or in the same species under various treatments, a method of estimating I should be of use to behavioral scientists.

2. The Distribution of $I(\hat{p})$ when Acts are Dependent.

For convenience, we will discuss the distribution of G^2 rather than $I(\hat{p})$, and define the analogous parameter

$$\begin{aligned}\lambda &= \lambda(p) = 2n \log_e 2 I(p) \\ &= 1.39n I(p).\end{aligned}$$

When n is large and $\lambda = I = 0$ (i.e., acts are independent), it is well-known (Wilks, 1938) that G^2 has approximately a chi-squared distribution with $(J-1)(K-1)$ degrees of freedom. The large sample distribution of G^2 (and thus of $I(\hat{p})$) is more complicated if λ (and thus $I(p)$) is positive. In general, the distribution of G^2 , even for n large, depends in a complicated way upon the p_{jk} , which are usually unknown. See Bishop, Fienberg, and Holland (1975, pp. 518-519) for a discussion and Broffit and Randles (1977) for a related discussion. However, if $I(p)$ is near 0, but not equal to 0, then it was proved by Wald (1943, Theorem IX), and under simpler assumptions by Feder (1968) and by Davidson and Lever (1970), that when n is large the distribution of G^2 is approximately that of a non-central chi-squared variable, with $(J-1)(K-1)$ degrees of freedom, and with noncentrality parameter equal to λ . (The references cited prove this for likelihood ratio tests in general. In the appendix it is shown that their definition of the non-centrality parameter reduces to our definition of λ in the present case. See Johnson and Kotz (1970) for a discussion of the non-central chi-squared distribution, which has mean $df + \lambda$ and variance $2df + 4\lambda$, if df is the degrees of freedom.) The meaning of the phrase " $I(p)$ near 0 and n large" is that one envisions a sequence of alternative hypotheses $p_{jk} \rightarrow p_j p_{.k}$, and sample sizes $n \rightarrow \infty$, such that the product $nI(p)$, or equivalently λ , approaches a fixed number. In any application, of course, there is only one value of n and $I(p)$, and the question then arises of how closely the noncentral chi-squared distribution approximates that of G^2 .

The adequacy of this approximation depends on two factors:

- 1) the size of np_{jk} , the expected cell frequencies, and
- 2) $|p_{jk} - p_j \cdot p_{\cdot k}|$, the cell deviations from independence.

The former quantities should be large; the latter, small. How large, and how small, depends on the degree of approximation desired, and to a lesser extent, on the other characteristics of the p_{jk} 's. Many rules of thumb have been suggested for how large the expected cell sizes should be for chi-squared tests of contingency tables to be valid. Most authorities would probably agree that if every cell in the observed table has at least 2 observations, the approximations are practically adequate. Also, if fewer than half the cells have at least two observations, it is certainly wrong to use this theory. In between these rather broad limits, the results should be interpreted with caution. When many of the cell sizes are too small, the low-frequency types of acts could be excluded, or combined if they group themselves into natural categories. As for the size of $|p_{jk} - p_j \cdot p_{\cdot k}|$, the simulation study presented in Table I suggests that if the cell sizes are not too small and if every $|p_{jk} - p_j \cdot p_{\cdot k}| \leq .1$, the noncentral chi-squared approximation to the distribution of G^2 is fairly good for the larger percentage points, though not so good for the lower tail.

Table I presents the results of a computer simulation of the distribution of G^2 for 5 tables (sets $\{p_{jk}\}$). Four of the

| True Table (p_{jk}) | I(p) (df) | Max $ p_{jk} - p_{j,k} $ | Sample Size n | Min n_{jk} | Noncentrality λ | Count of times $G^2 \leq \chi^2_{df, \alpha}(\lambda)$ in 1000 simulations | | | | |
|----------------------------|--------------|--------------------------|------------------|--------------|----------------------------|--|--------------|--------------|--------------|--------------|
| | | | | | | 25 ± 1.0 | 100 ± 20 | 500 ± 32 | 900 ± 20 | 975 ± 10 |
| T_1 | .3 | .2 | .029 | .05 | 20 | 4 | .804 | 41* | 104 | 545* |
| | .2 | .3 | (1) | | 100 | 20 | 4.02 | 32 | 108 | 497 |
| T_2 | .4 | .1 | .278 | .15 | 20 | 2 | 7.71 | 22 | 76* | 437* |
| | .1 | .4 | (1) | | 100 | 10 | 38.5 | 18 | 75* | 467* |
| T_3 | .05 | .05 | .090 | .04 | 20 | 1 | 2.50 | 217* | 289* | 476 |
| | .05 | .85 | (1) | | 100 | 5 | 12.5 | 60* | 136* | 509 |
| T_4 | .08 | .02 | .258 | .07 | 20 | .4 | 7.15 | 178* | 183* | 402* |
| | .02 | .88 | (1) | | 100 | 2 | 35.8 | 53* | 139* | 493 |
| T_5 | .25 T_1 | .25 T_3 | .458 | .10 | 20 | .1 | 12.7 | 21 | 85 | 539* |
| | .25 T_4 | .25 T_2 | (9) | | 100 | .5 | 63.4 | 15 | 77* | 482 |
| | | | | | 500 | 2.5 | 317 | 16 | 93 | 481 |
| | | | | | 2500 | 12.5 | 1586 | 21 | 95 | 505 |
| | | | | | | | | | | 984 |

TABLE I. A Computer simulation of the distribution of G^2 for 5 sets $\{p_{jk}\}$.

*Differs significantly (2 standard errors) from that expected by the noncentral chi-squared distribution.

tables, T_1, \dots, T_4 have $J = K = 2$, and the p_{jk} are given in the column labeled "True Table" of Table I. Notice that T_1 and T_2 have marginal frequencies of 50-50 for the two types of act, while T_3 and T_4 have a more unbalanced 90-10 marginal distribution. Table T_5 is a larger, 4×4 , table formed by dividing the cell probabilities of T_1 through T_4 by 4, and arranging them as indicated. The next columns of Table I contain the information measure $I(p)$, the largest value of $|p_{jk} - p_{j.}p_{.k}|$, the simulated sample size n , the smallest expected cell size, and the noncentrality parameter, $\lambda = 2n(\log_e 2)I(p)$, for each simulated experiment. For each combination of n and $\{p_{jk}\}$, 1000 random samples from a simple multinomial (neither margin fixed) were taken, and the value of G^2 computed. Computer generated uniformly-distributed pseudo-random numbers were used to generate the samples, computed using a multiplicative congruential generator. (See Ericson and Fox, 1976, pp. 66-69.) The last five columns of Table I give the empirical cumulative distribution of G^2 , namely the number of times (out of 1000) that the observed G^2 was less than or equal to the 2.5, 10, 50, 90, and 97.5 percentiles of the noncentral chi-squared distribution with the corresponding df and λ . The table entries marked with an asterisk are those values of the observed cdf of G^2 which differed significantly (2 standard errors) from that of the

noncentral chi-squared distribution. As was mentioned above, the upper tails of the observed distributions of G^2 agree well with approximation whenever there are 2 or more expected in each cell, and every $|p_{jk} - p_{j.} p_{.k}| \leq .1$. This suggests that approximate lower confidence limits for λ , and thus $I(p)$, can profitably be found, using percentiles of the noncentral chi-squared distribution. Since Table I shows that the lower tails of G^2 are not so well approximated for these sample sizes, it seems that rather larger expected cell frequencies, and/or smaller values of $|p_{jk} - p_{j.} p_{.k}|$ are needed in order to trust upper confidence limits for λ . Fortunately, in most applications one is more interested in establishing lower limits for $I(p)$.

3. Obtaining Approximate Confidence Limits for $I(p)$.

If the noncentral chi-squared approximation to the distribution of G^2 is accurate, and if we let $\chi^2_{df, \alpha}(\lambda)$ be the α^{th} percentile of the distribution with degrees of freedom $df = (J - 1)(K - 1)$ and noncentrality λ , then if the value G^2 is observed, the solution to

$$\chi^2_{df, \alpha}(\lambda) = G^2$$

is an approximate one-sided confidence limit for the true λ ,

with confidence coefficient α for an upper bound. This is converted to a bound on the information $I(p)$ by the relation

$$I(p) = \lambda/1.39n.$$

If either df or λ is quite large, then Johnson (1959) shows that $(\chi^2 - df - \lambda + 1)/(2\chi^2 + 2\lambda)^{1/2}$ is approximately a standard normal deviate. This leads easily to the following approximate confidence limits λ_α :

$$\lambda_{.5} = G^2 - df + 1$$

$$\lambda_\alpha = \lambda_{.5} + z_\alpha^2 + z_\alpha [2(G^2 + \lambda_{.5}) + z_\alpha^2]^{1/2}$$

where z_α is the α^{th} percentile of the standard normal distribution. For df and λ only moderately large it is necessary to solve the equation $\chi^2_{df, \alpha}(\lambda) = G^2$ directly, and Figures A and B are designed to do this easily, for $\alpha = 2.5\%$ and 97.5% respectively. One merely finds the value of G^2 on the abscissa, goes up vertically to the curve labeled with appropriate degrees of freedom, interpolating with respect to df if necessary, and then reads the value of λ_α on the ordinate scale. Figure A is used for an upper limit and

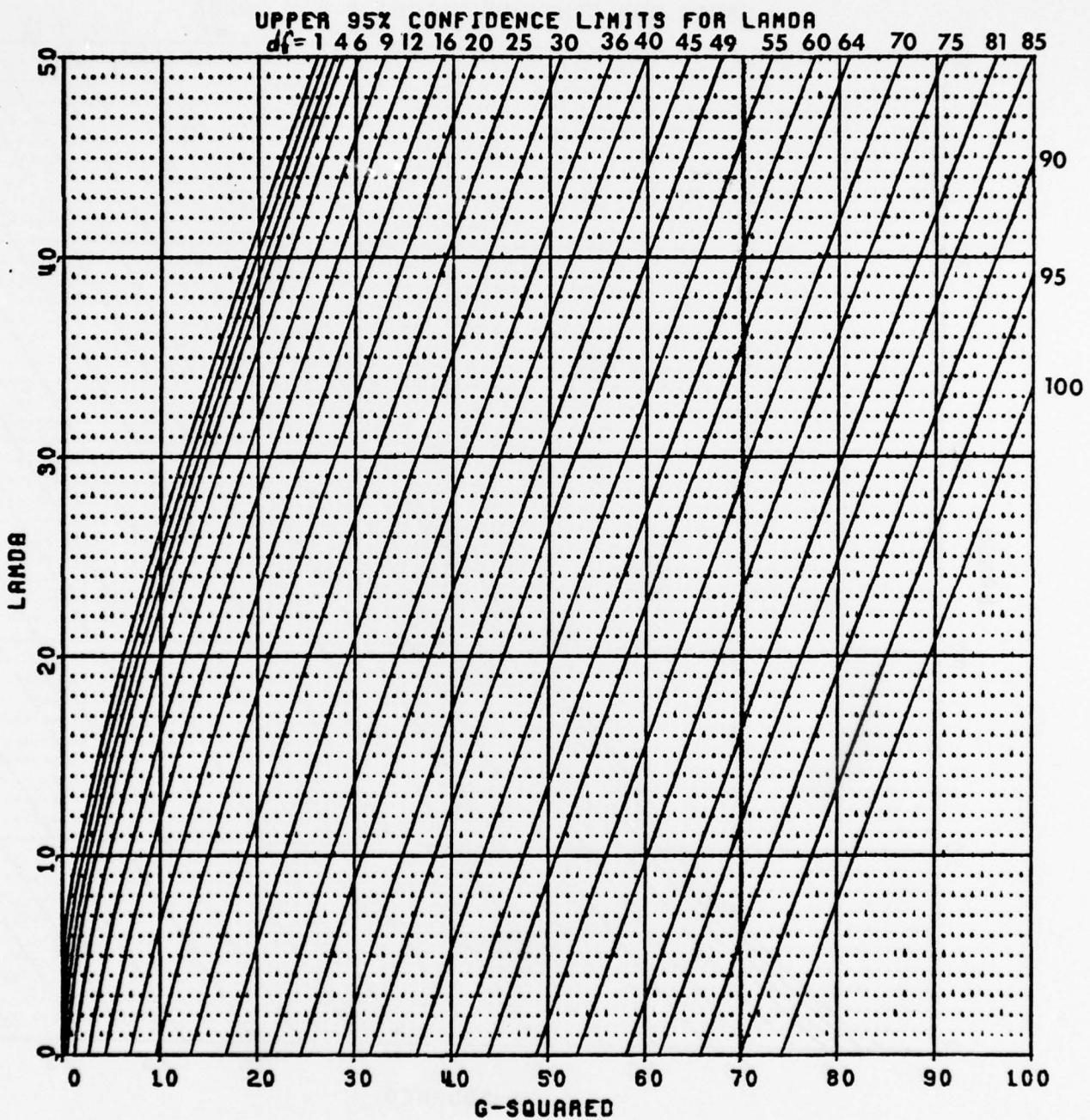


Figure A. 95% confidence intervals for the noncentrality λ , if G^2 is an observation from a noncentral chi-squared $\chi^2_{df}(\lambda)$ distribution: upper endpoints.

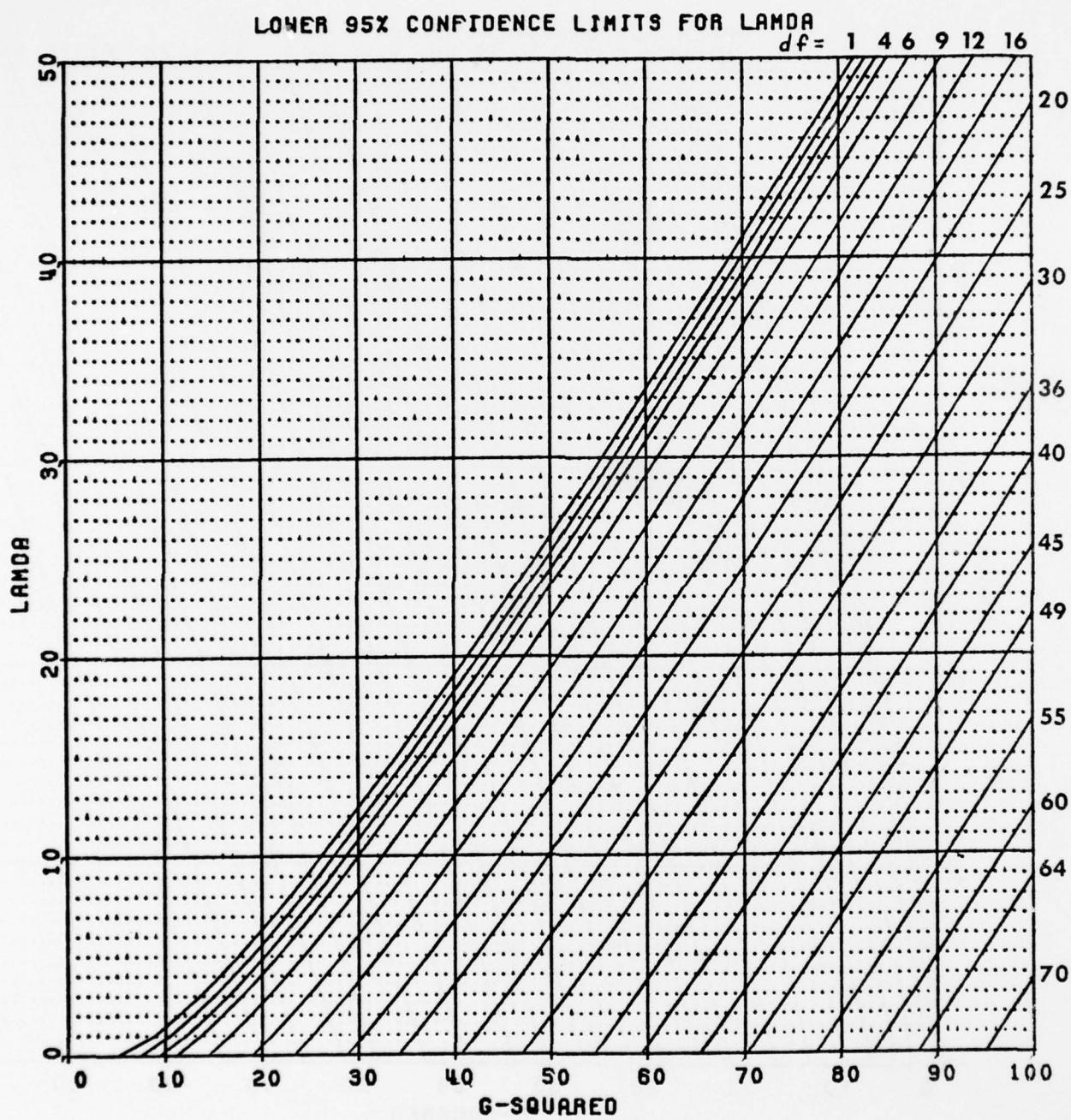


Figure B. 95% confidence intervals for the noncentrality λ , if G^2 is an observation from a noncentral chi-squared $\chi^2_{df}(\lambda)$ distribution: lower endpoints.

Figure B for a lower limit. The two limits together form an approximate 95% confidence interval for λ and are converted to an interval for $I(p)$ upon division by $1.39n$. The noncentral chi-squared percentiles displayed in the figures were computed using Pearson's (1959) approximation for most values of λ and df , and a series expansion for the distribution function (see Johnson and Kotz, 1970, vol. 2, p.132) in the few cases where Pearson's approximation was not accurate enough for the figures.

If a point estimate of $I(p)$ is desired, one may take $\hat{I} = \lambda_5/1.39n$, which is approximately unbiased for large n , and has approximate standard error $[2df + 4\lambda]^{1/2}/1.39n$. If n is large, the difference between \hat{I} and $I(\hat{p})$ will not be important. However, if n is not large compared to df , then the fact that \hat{I} "corrects" for df can be important, at least in terms of mean square error (MSE) of the estimator. For example, in the simulations, for the 4×4 Table T_5 (with $df = 9$, $n = 20$, and $I(p) = .458$), the 1000 simulated G^2 's had a sample mean of 20.5 and sample standard deviation of 6.4. Simple calculations then show that \sqrt{MSE} was .36 for $I(\hat{p})$ but only .23 for \hat{I} . The \sqrt{MSE} would be even smaller for the estimate defined to be $\max(0, \hat{I})$, which is of course the estimate one would report in practice. Often it is desired to compare values of $I(p)$ for different populations. A simple intuitive test is to notice whether confidence intervals for

the separate population parameters overlap. Although this criterion will be more conservative than a proper likelihood ratio test for $I(p_1) = I(p_2)$, it is much simpler. If one desires to estimate the common value of $I(p)$ for two or more populations, one can use the present methods after adding the corresponding values of G^2 , df, and n from the various samples. This is practically equivalent to averaging the various values of \hat{I} , weighted by their sample size. However this may not be as efficient as the much more complicated maximum likelihood procedure (see Johnson and Kotz, 1970, p. 136).

4. Applications to the Literature

Some examples of the application of the distribution theory discussed above to experimental data are presented in Table II and will now be discussed. It should be noted that the use of information statistics or a chi-squared analysis is not strictly appropriate for most of these data, since there are often too many cells with low expected values. However, published tables with sufficiently high expected values that depict information transfer in animals are rare, and so we have decided to perform the analyses on these data. Our results must certainly be more accurate than the heretofore uniform practice of ignoring the variability of G^2 and merely assuming $I(p) = I(\hat{p})$.

(Actually, only the Altmann (1964) data, discussed next, flagrantly violate the rules of thumb established in section 2.)

| Line | $\hat{t}(p)$ | \hat{t} | Confidence Limits | Species | Source | Remarks | n | df | Line |
|------|--------------|-----------|----------------------|------------------------------|---|-----------------------------|------|-------|------|
| 1 | 2.01 | .08 | .03-.13 | Rhesus | Altmann (1965 and personal communication) | | 4571 | 12210 | 1 |
| 2 | 1.18 | 1.14 | 1.08-1.19 | " | Altmann's data grouped (see text) | | 4571 | 289 | 2 |
| 3 | .36 | .18 | .13-.24 | Pylopogurus operculatus | Hazlett & Bossert (1965) | between indiv. | 1030 | 256 | 3 |
| 4 | .35 | .29 | .25-.33 | Clibanarius tricolor | " | " | 2000 | 169 | 4 |
| 5 | .39 | .27 | .22-.34 | Calcinus tibicen | " | " | 1040 | 169 | 5 |
| 6 | .52 | .36 | .30-.43 | Pagurus marshi | " | " | 1060 | 225 | 6 |
| 7 | .63 | .53 | .43-.65 | Gonodactylus bredini | Dingle (1969) | 1st 10 min-- between indiv. | 523 | 72 | 7 |
| 8 | 1.03 | .88 | .72-1.07 | " | " | 2nd 10 min. | 355 | 72 | 8 |
| 9 | .78 | .65 | .51-.80 | " | " | middle 20 min. | 393 | 72 | 9 |
| 10 | .7 | .53 | .38-.70 | " | " | last 20 min. | 272 | 64 | 10 |
| 11 | .58 | .51 | .41-.64 | " | " | 1st 10 min-- within indiv. | 447 | 42 | 11 |
| 12 | .79 | .75 | .63-.90 | " | " | last 50 min. | 690 | 42 | 12 |
| 13 | .40 | .27 | .18-.45 | Gonodactylus spinulosus | Dingle (1972) | 1st 10 min-- within indiv. | 235 | 42 | 13 |
| 14 | .64 | .52 | .04-.65 | " | " | 1st min-- between indiv. | 427 | 72 | 14 |
| 15 | .5 | .38 | .29-.50 | G. spinulosus vs. G. bredini | " | 1st min-- between indiv. | 446 | 72 | 15 |

Table II. Application of the method for confidence limits to examples in the literature.

Altmann, in his 1965 study of social communication in rhesus monkeys on Cayo Santiago Island, reports on the amount of information held in common between adjacent acts (line 1). This number, $I(\hat{p}) = 2.01$ bits*, is larger than that reported for any other species except man. Altmann notes that his study is beset by a number of methodological difficulties, including the fact that he did not distinguish between intra- and inter-individual sequences and the fact that he did not distinguish between various kinds of behavior, such as maternal, courtship, aggressive, etc. From the point of view of assessing the significance of Altmann's findings, another difficulty is the fact that he recognized 120 possible types of acts. The results is a two-way table with 14 400 cells but only 4 571 observations! A formal application of our procedure yields $\hat{I} = .08$ with a 95% confidence interval for $I(p)$ of from .03 to .13. The degrees of freedom used in the computation is $df = 12 210$ and reflects the fact that some rows and columns of the observed table are entirely empty (and thus omitted) because some rare acts occurred only at the beginnings or ends of sequences. However, before this data could be used to support or reject Altmann's contention that rhesus monkeys do communicate, one must either develop a theory for the distribution of $I(\hat{p})$ when most cells are empty, or combine similar types of acts to reduce the degrees of freedom and increase the cell sizes.

Using as a guide the groups of acts suggested by Altmann (1965, Table 2) and using the data kindly provided by Professor

*Actually Altmann (1965) reports 1.96 bits per act, but a recomputation of his figure (using data kindly supplied to us by Professor Altmann) with a larger computer yields the figure 2.01. The difference between the two values is negligible for our purposes.

Altmann, we lumped the 120 acts into 18 groups. Groups 1-13 each contained a single act, namely all those which occurred 98 times or more and originally numbered 2-5, 9, 19, 25, 41, 43, 45, 50, 51, and 99. The remaining acts were placed into one of the 5 groups 14-18, with rough meanings as follows:

group 14: fear, submission and compound acts containing
fearful and submissive elements

group 15: attack, threat, and their compounds

group 16: friendly acts and compounds

group 17: vocalizations

group 18: miscellaneous -- other acts hard to group logically.

This grouping has no doubt lumped some dissimilar acts, especially in groups 17 and 18, but our purpose here is not to propose a particular set of group definitions for this data, but merely to demonstrate the effectiveness of grouping in general when estimating $I(p)$. The resulting 18×18 table has 324 cells, of which 95 have $n_{jk} \geq 10$, 100 cells have $2 \leq n_{jk} \leq 9$, and 129 have $n_{jk} = 0$ or 1. Every value of $|\hat{p}_{jk} - \hat{p}_j \cdot \hat{p}_{.k}|$ is less than .04. The rules of thumb given in section 2 suggest using our approximate theory with a cautious interpretation. The results are given in line 2 of Table II: $I(\hat{p}) = 1.18$, $\hat{I} = 1.14$, and $1.08 < I(p) < 1.19$ with 95% confidence. Thus we find evidence to support the value 1.1 bits per act for communication among rhesus monkeys, but not the value 2.0 reported by Altmann (1965).

Hazlett and Bossert (1965) present data on aggressive communication in a number of hermit crabs, some of which is presented in lines 3-6. They assert that, while Clibanarius tricolor and Calcinus tibicen have roughly the same value for the information parameter, that for Pagurus marshi is higher. Hazlett and Bossert then provide plausible biological reasons why information transfer is more essential to Pa. marshi than Cl. tricolor or Ca. tibicen. Although our analysis does not contradict these explanations, inspection of the confidence intervals shows that there is no strong evidence differentiating these three species, although $I(p)$ for Py. operculatus is quite surely below that of Pa. marshi. Hazlett (personal communication) has been unable to suggest an interpretation of this result.

In Dingle's (1969) experiments of aggressive communication in the mantid shrimp, Gonodactylus bredini, two adults of the same sex were placed in a finger bowl and allowed to interact for one hour. Data obtained from twenty such pairings were lumped together. Each 60 minutes of data were broken into two 10-minute and two 20-minute periods. A separate transition matrix was formed for each period. To test stationarity of the process, column marginals of temporally adjacent tables were compared using a chi-squared test, revealing significant differences except between the two 20-minute tables. Dingle asserted that information transferred during the second 10-minute period is significantly higher than any of the other periods. Furthermore, this dependence of the information statistic on time was also found in Gonodactylus spinulosus and in matches

between G. brendini and G. spinulosis (not shown). Dingle also found a decline in the frequency of aggressive acts in G. brendini, and a difference in the types of transitions as time went on. He attributes these effects to the establishment of a dominant-subordinate relation in the species mentioned during the second 10 minutes.

Our analysis of the four time periods, which is presented in lines 7-10 of table II, shows that information transferred during the second period is indeed significantly higher than during either the first or the last period, but not significantly different from the third.

Dingle also compiled tables depicting transitions between each individual's act, and the next act of the same animal. It should be noted that this type of intra-individual analysis confounds two-step dependence with two steps of one-step dependence (Oden, 1977). However, we have calculated confidence intervals for these data and present them in lines 11,12 and, for G. spinulosis, 13

In calculating the information statistic for G. spinulosis and matches between G. spinulosis and G. brendini, Dingle (1972) followed essentially the same protocol as for G. brendini. The results are reported in lines 13-15.

Dingle observed that, generally, values for mantid shrimp exceed those for hermit crabs. He suggests that this might reflect the greater seriousness of physical combat in the mantid shrimp.

Although the two independent values of \hat{I} between individual mantid shrimps (lines 7 and 14) are each higher than all 4 \hat{I} 's from experiments with hermit crabs (lines 3 - 6), this in itself is not much support for Dingle's interpretation, since a Mann-Whitney U-test on the ranks of the \hat{I} or $I(\hat{p})$ statistics for mantid shrimp and hermit crabs fails to reveal a significant difference between the two classes ($\alpha = .13$, two-tailed). However, the fact that there is only one overlap in the 95% confidence intervals between the two groups (G. spinulosis, line 14, and P. marshi, line 6), while there is much overlap within the hermit crabs, tends to confirm his assertion.

5. Acknowledgements

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APPENDIX

The noncentral chi-squared approximation to the distribution of the likelihood ratio statistic.

This appendix will show that the definition of the non-centrality parameter λ in Wald's (1943) proof of the asymptotic distribution of the likelihood ratio statistic is the same, for n large and $I(p)$ small, as $\lambda(p) = 2n \log_e 2 I(p)$, where

$$I(p) = \sum_{j,k} p_{jk} \log_2 p_{jk} / p_j \cdot p_k .$$

Let X be a discrete-valued random vector with probabilities $p(x; \theta)$. The parameter θ belongs to a subset Θ of Euclidian space E^q and is unknown. Let $g(\cdot)$ be a function over the parameter space which takes values in E^r ($r \leq q$) and has continuous first partial derivatives. To test the hypothesis $g(\theta) = 0$ versus $g(\theta) \neq 0$ using a sample of n independent observations of X , consider the likelihood ratio statistic

$$G^2 = 2 \log \left[\sup_{\theta \in \Theta} \prod_{i=1}^n p(x_i; \theta) \right] / \left[\sup_{g(\theta)=0} \prod_{i=1}^n p(x_i; \theta) \right]$$

Suppose $\{\theta_n\}$ is a sequence of alternatives converging at rate $n^{-1/2}$ to a point θ_0 such that $g(\theta_0) = 0$ and the matrix $\partial g(\theta_0) / \partial \theta$ is of full rank, i.e., $\theta_n = \theta_0 + C n^{-1/2}$ for some fixed vector C . Then Wald's (1943) theorem IX states that the limiting distribution of G^2 is the noncentral chi-squared distribution with r degrees of freedom and non-centrality

$$(1) \quad \lambda = \lim_{n \rightarrow \infty} n[g(\theta_n)]' \Sigma^{-1}(\theta_n) [g(\theta_n)] ,$$

where $\Sigma(\theta)$ is the asymptotic covariance matrix of the quantity $n^{1/2}[g(\hat{\theta}) - g(\theta)]$ as $n \rightarrow \infty$ with θ fixed and $\hat{\theta}$ denoting the maximum likelihood estimator of θ based on sample size n .

Now the quantity

$$(2) \quad \delta(\theta_n) = [g(\theta_n)]' \Sigma^{-1}(\theta_n) [g(\theta_n)]$$

may be interpreted as the squared distance, in E^q , between the point θ_n and the nearest point on the surface $\{\theta: g(\theta) = 0\}$, where the metric used to define distance is that given by the Fisher information matrix $H(\theta_n)$, defined by

$$H(\theta) = E[(\partial \log p(x; \theta) / \partial \theta)(\partial \log p(x; \theta) / \partial \theta)']$$

$H^{-1}(\theta)$ is the asymptotic covariance matrix of $n^{1/2} \hat{\theta}$, and $\Sigma(\theta) = \dot{g} H^{-1}(\theta) \dot{g}'$, where $\{\dot{g}_{\ell m}\} = \{\partial g_{\ell}(\theta) / \partial \theta_m\}$ $\ell = 1, \dots, r$ and $m = 1, \dots, q$. It is well known (see e.g., Kullback (1959, p. 26-28) that Fisher information is a limiting form of the Kullback-Leibler information measure: namely for two points θ_0, θ_1 close together, $(\theta_1 - \theta_0)' H(\theta_1) (\theta_1 - \theta_0) = 2 \sum p(x; \theta_1) \log p(x; \theta_1) / p(x; \theta_0) + O(|\theta_1 - \theta_0|^3)$. Thus, if $g(\theta_0) = 0$, the minimum distance $\delta(\theta_n)$ is

$$(3) \quad \delta(\theta_n) = \min_{\{\theta : g(\theta) = 0\}} 2 \sum_x p(x; \theta_n) \log_e p(x; \theta_n) / p(x; \theta) + o(|\theta_n - \theta_0|^3).$$

In our case, where $x = (j, k)$, $p(x; \theta) = p_{jk}$, and $\{\theta : g(\theta) = 0\} = \{p_{jk} : p_{jk} - p_j \cdot p_{\cdot k} = 0\}$ it can be checked that the sum on the right of (3) is minimized by $p_{jk} = p_j^{(n)} p_{\cdot k}^{(n)}$

so that

$$\delta(\theta_n) = \delta(\{p_{jk}^{(n)}\}) = 2 \sum_{jk} p_{jk}^{(n)} \log_e p_{jk}^{(n)} / p_j^{(n)} p_{\cdot k}^{(n)} + o(n^{-3/2}),$$

assuming that $p_{jk}^{(n)} = p_{jk}^{(0)} + c_{jk} n^{-1/2}$, and that $p_{jk}^{(0)} - p_j^{(0)} p_{\cdot k}^{(0)} = 0$ for all j, k . Thus, combining (1) - (4), we have $\lambda = \lim_{n \rightarrow \infty} 2 n (\log 2) I(p^{(n)})$, as desired.

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A much-used measure of dependence in two-way tables is

$I(p) = \sum_j \sum_k p_{jk} \log_2(p_{jk}/p_j \cdot p_{\cdot k})$. It is shown that, for large n and small $I(p)$, estimation of $I(p)$ is equivalent to estimating λ , the noncentrality parameter of the noncentral chi-squared distribution. Methods for calculating estimates and confidence limits for $I(p)$ are presented. Monte Carlo simulation suggests rule-of-thumb meanings for 'large n ' and 'small $I(p)$ ', and shows that the lower confidence limits are more accurate than the upper ones. Applications to several previously published studies of animal behavior are presented.

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